Abstract

A new inverse feedback structure for adaptive control of periodic noise is introduced for systems with nonminimum phase cancellation path. To obtain the inverse model of the nonminimum phase cancellation path, the cancellation path model can be factorized into a time-delay term, a minimum phase term and a maximum phase term. The maximum phase term containing unstable zeros makes the inverse model unstable. To avoid the instability, we alter the inverse model of the maximum phase system into an anti-causal FIR one. An LMS predictor estimates the future samples of the noise, which are necessary for causality of both anti-causal FIR approximation for the stable inverse of the maximum phase system and time-delay existing in the cancellation path. The proposed method has a faster convergence behaviour and a better transient response than the conventional filtered-x LMS algorithms with the same internal model control structure since a filtered reference signal is not required. We compare the proposed methods with the conventional methods through simulation studies.

INTRODUCTION

An effort to cancel the noise by superposing a sound with the opposite phase of the noise, which is called the active noise control (ANC), has been widely studied. ANC
is very effective especially against low frequency noise below 500 Hz, which is hard to reduce by passive means. Active control of periodic noise has received a great deal of attention because most rotating machinery such as an engine and a fan generates periodic noise [1-3]. The filtered-x LMS algorithm based on the internal model control (IMC) [4] is the most famous among various methods of adaptive feedback control, which requires the error signal alone to attenuate periodic noise. This method has a slow convergence speed and an unwanted transient response such as overshoot [3]. To overcome this problem Bouchard [3] proposed an inverse feedback structure. However, the nonminimum phase cancellation path and the effect of the measurement noise, which made the performance deteriorated, were not considered.

In this paper, an adaptive inverse feedback control method of time-varying periodic noise is suggested for systems with nonminimum phase cancellation path. The method follows the two steps. One is a direct inversion of nonminimum phase cancellation path in off-line manners and the other is a LMS predictor in on-line manners, which is required to predict the future data of the periodic noise because the inverse obtained in the first step is a delayed version of the exact inverse. Computer simulations for two kinds of the conventional methods and the suggested methods are carried out under various conditions.

APADTIVE FEEDBACK CONTROL OF PERIODIC NOISE

Active Control of Periodic Noise Using IMC Technique

An active feedback control method based on IMC technique [4] using a filtered-x LMS algorithm, which is a feedforward algorithm, does not measure the reference signal but estimate the reference signal using IMC technique. Figure 1 shows the block diagram of ANC using IMC technique. The desired signal \( d(n) \) (unwanted noise) is estimated from the error signal \( e(n) \) and is used as a reference signal \( r(n) \) of a filtered-x LMS algorithm. \( H \) is a cancellation path, \( \hat{H} \) is an estimate of the
cancellation path and $W$ is a controller filter. The system has the best performance when the reference signal $r(n)$ is the same as the desired signal $d(n)$. Background noise such as measurement noise included in $r(n)$ makes the performance deteriorated. The filtered-$x$ LMS algorithm requires the reference signal filtered through the cancellation path and has drawbacks as follows: slow convergence speed and unwanted transient response such as overshoot [3].

**Active Control of Periodic Noise Using Frequency Estimation**

![Figure 2. ANC using frequency estimation](image)

An active feedback control method based on noise frequency estimation [2] also exploits the filtered-$x$ LMS algorithm. The frequencies of the noise are estimated by using the constrained notch filter [2]. Figure 2 shows block diagram of ANC using the frequency estimation. The method uses the multiple sinusoids generated by estimating the frequency of the noise as the reference signal. If the estimated frequencies are exact the method has better performance than the ANC method based on IMC technique because the generated reference of the method does not include the measurement noise. However, the method also uses the filtered-$x$ LMS algorithm and has the same drawbacks as the ANC based on IMC technique.

**ADAPTIVE INVERSE FEEDBACK CONTROL OF PERIODIC NOISE**

**Direct Inversion of Nonminimum System**

An inversion of the nonminimum phase system inherently results in an unstable system. We approximate the inverse of the nonminimum phase system to a form of the stable anti-causal FIR filter. The nonminimum phase system can be separated into time delay, minimum phase and maximum phase sub-systems. The maximum phase sub-system makes the inverse system unstable and is assumed to have $l$ unstable zeros as follows:

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\[ H(z) = z^{-\Delta} H_m(z) H_u(z) \]  
\[ H_u(z) = \prod_{i=1}^{l} (1-b_i z^{-1}), \quad |b_i| > 1, \forall i \]  
\[ H_u(z)^{-1} = \prod_{i=1}^{l} \frac{1}{1-b_i z^{-1}} = \sum_{i=1}^{l} c_i \frac{1}{1-b_i z^{-1}} \equiv \frac{x(n)}{y(n)}, \]  
\[ c_i = (1-b_i z^{-1}) H_u(z)^{-1} \big|_{z=b_i}, i = 1, 2, \ldots, l \]  
\[ x(n) = -l \sum_{i=1}^{M_i} \sum_{k=1}^{b_i} \frac{c_i}{b_i^{M_i}} y(n+k) - \sum_{i=1}^{l} \frac{1}{b_i^{M_i}} x(n+M_i) \approx \sum_{i=1}^{l} \sum_{k=1}^{b_i} \frac{c_i}{b_i^{M_i}} y(n+k). \]  
\[ H_u(z)^{-1} \approx -l \sum_{i=1}^{M_i} c_i \frac{z^{-k}}{b_i^{M_i}}. \]  
\[ H(z)^{-1} \approx z^{\Delta+M} H_m(z)^{-1} \tilde{H}_u(z)^{-1} \text{ for } M = \max(M_i), \forall i \]  
\[ \tilde{H}_u(z)^{-1} = -l \sum_{i=1}^{M_i-1} \sum_{k=0}^{b_i^{M_i-k}} c_i z^{-k} \]  

The inverse of the maximum phase system can be expressed as equation (3). If input and output of the inverse system are defined as \( y(n) \) and \( x(n) \), respectively the stable output \( x(n) \) can be obtained as in equation (5) if \( M_i \) is large enough. Equation (4) represents a residue value for partial fraction expansion. The inverse of the maximum phase system can be approximated to an anti-causal FIR filter as in equation (6). The inverses of the minimum phase system and time delay term can be easily obtained and consequently, the inverse of the nonminimum phase system can be approximated as in equation (7).

![LMS predictor](image)

Figure 3. LMS predictor
Here, LMS predictor is used to predict the future samples because the inverse is anti-casual. Figure 3 shows the block diagram of LMS predictor [5]. We suggest an adaptive inverse feedback structure using the direct inversion method of the nonminimum phase system in the next section.

**Adaptive Inverse Feedback Structure**

![Block diagram of LMS predictor](image)

**Figure 4. Adaptive inverse feedback control method I**

**Figure 5. Adaptive inverse feedback control method II (enhanced)**

Design of an adaptive inverse feedback control for systems with nonminimum phase cancellation path can be divided into two steps: One is a stable inversion of nonminimum phase cancellation path in off-line manners and the other is a LMS predictor design in on-line manners. Because the suggested method is updated by LMS algorithm it has better performance in view of convergence speed and transient response than the conventional method using filtered-x LMS algorithm. Figure 4 shows block diagram of the proposed method.

In case of low signal-to-noise ratio (SNR), the performance of LMS predictor might be deteriorated by the measurement noise. An adaptive line enhancer using an
adaptive notch filter [2] can make the periodic component of the contaminated desired signal enhanced and increases the performance of LMS predictor. Figure 5 shows block diagram of the method.

**COMPUTER SIMULATIONS**

Computer simulations for two kinds of the conventional methods (Method 1, Method 2) and two kinds of the proposed methods (Method 3, Method 4) were carried out under various conditions: simulations for the time-invariant periodic noises contaminated with white noise of 10dB and 0dB SNRs and for the time-varying periodic noise which consisted of two pure tones.

- Method 1: ANC using IMC technique
- Method 2: ANC using frequency estimation
- Method 3: Adaptive inverse feedback control I – proposed
- Method 4: Adaptive inverse feedback control II (enhanced) – proposed

\[
H(z) = \frac{z^{-6}(1-0.9891z^{-1})(1-6.4337z^{-1})}{(1-(0.6564 + i0.6343)z^{-1})(1-(0.6564 - i0.6343)z^{-1})} \tag{9}
\]

\[
\tilde{H}_u(z)^{-1} = -\frac{1}{6.4337^4} - \frac{1}{6.4337^3} z^{-1} - \frac{1}{6.4337^2} z^{-2} - \frac{1}{6.4337} z^{-3} \tag{10}
\]

\[
d(n) = 2\sin(2\pi \cdot 150 \cdot nT) + 1.5\sin(2\pi \cdot 200 \cdot nT + 1) + \xi(n) \tag{11}
\]

A cancellation path is a nonminimum phase system as in equation (9). A maximum phase subsystem can be approximated to equation (10) using equation (8). Equation (11) represents time-invariant periodic noise contaminated by white noise.

**Table 1. Performance comparison (SNR = 10dB)**

<table>
<thead>
<tr>
<th></th>
<th>Method 1</th>
<th>Method 2</th>
<th>Method 3</th>
<th>Method 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filter length</td>
<td>20</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Step size</td>
<td>0.07</td>
<td>0.07</td>
<td>0.1</td>
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<tr>
<td>OSPL reduction [dB]</td>
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<td>-9.2</td>
<td>-7.5</td>
<td>-10.1</td>
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<tr>
<td>Peak reduction [dB]</td>
<td>-30,-30</td>
<td>-30,-30</td>
<td>-30,-25</td>
<td>-30,-30</td>
</tr>
<tr>
<td>Convergence step</td>
<td>1200</td>
<td>500</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>Overshoot [ratio]</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

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Table 2. Performance comparison (SNR = 0dB)

<table>
<thead>
<tr>
<th>SNR = 0dB</th>
<th>Method 1</th>
<th>Method 2</th>
<th>Method 3</th>
<th>Method 4</th>
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<tbody>
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<td>Filter length</td>
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<td>20</td>
<td>20</td>
<td>10</td>
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<tr>
<td>Step size</td>
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<td>0.02</td>
<td>0.02</td>
<td>0.2</td>
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<tr>
<td>OSPL reduction [dB]</td>
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<td>-2.8</td>
<td>-2.2</td>
<td>-2.8</td>
</tr>
<tr>
<td>Peak reduction [dB]</td>
<td>-20,-10</td>
<td>-30,-15</td>
<td>-20,-10</td>
<td>-30,-15</td>
</tr>
<tr>
<td>Convergence step</td>
<td>1200</td>
<td>1000</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>Overshoot [ratio]</td>
<td>1.5</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

(a) Method 1
(L = 10, \( \mu = 0.01 \), R = -5dB)

(b) Method 2
(L = 10, \( \mu = 0.1 \), R = -17dB)

(c) Method 3
(L = 10, \( \mu = 0.3 \), R = -28dB)

(d) Method 4
(L = 10, \( \mu = 0.3 \), R = -23dB)

Figure 6. Signal before and after control
(−−: before control, →: after control, L: tap, \( \mu \): step size, R: error reduction)
Tables 1 and 2 show the simulation results for periodic noise of equation (11). In tables ‘OSPL’ represents overall sound pressure level and ‘Overshoot [ratio]’ represents magnitude ratio of before and after control in transient responses. Methods 3 and 4 have a fast convergence speed and no overshoot in transient response. Methods 2 and 4 have good reduction performance in low SNR.

Figure 6 shows the results for time-varying periodic noise without contamination. The periodic noise consists of two tones with linearly time-varying frequency 150Hz ~ 230Hz and 200Hz ~ 320Hz in 3 seconds, respectively. In case of methods 1 and 2 step size should be large enough for fast convergence and good performance but it induced overshoot in transient response. Therefore, we properly selected it in trade-off sense. Methods 3 and 4 have good performance due to fast convergence speed. Error signal of method 4 was slightly larger than that of method 3 because of the bias estimation of the frequencies of the time-varying noise.

**CONCLUSIONS**

New inverse feedback structures for adaptive control of periodic noise for systems with nonminimum phase cancellation path are proposed. The methods have faster convergence speed and better transient responses than the conventional methods based on the filtered-x LMS algorithm. Especially, the proposed methods are very efficient in reducing time-varying periodic noise.

**REFERENCES**